

(a) Intermediate Value Theorem

If $f(x)$ is continuous on a closed interval $[a, b]$ and c is any number between $f(a)$ and $f(b)$, then there is at least one number x in the closed interval such that $f(x) = c$.

(b)

$f(x)$ is continuous on $[0, 1]$ since it is continuous everywhere.

$$f(0) = 2$$

$$f(1) = -3$$

$$-3 < 0 < 2$$

$$f(1) < 0 < f(0)$$

By the Intermediate Value Theorem, there is at least one number x in $[0, 1]$ such that $f(x) = 0$.

So, $f(x)$ has a zero in $[0, 1]$.